Analysis of the Neutralino System in Two–Body Decays of Neutralinos

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Abstract

In the minimal supersymmetric standard model (MSSM), the neutralinos, the spin–1/2 Majorana superpartners of the neutral gauge and Higgs bosons, are expected to be among the light supersymmetric particles that can be produced copiously at future high–energy colliders. We analyze two–body neutralino decays into a neutralino plus a Z boson or a lightest neutral Higgs boson h, allowing the relevant parameters to have complex phases. We show that the two–body tree–level decays of neutralinos are kinematically allowed in a large region of the MSSM parameter space and they can provide us with a powerful probe of the Majorana nature and CP properties of the neutralinos through the Z–boson polarization measured from Z–boson leptonic decays.

1 Introduction

The search for supersymmetry (SUSY) is one of the main goals at present and future colliders [1] as SUSY is generally accepted as one of the most promising concepts for physics beyond the Standard Model (SM) [2]. A special feature of SUSY theories is the existence of the neutralinos, the spin–$1/2$ Majorana superpartners of the neutral gauge bosons and Higgs bosons. In the MSSM, the neutralinos are expected to be among the light supersymmetric particles that can be produced copiously at future high–energy colliders [3]. Once several neutralino candidates are observed at such high–energy colliders, it will be crucial to establish the Majorana nature and CP properties of the neutralinos. In this light, many extensive studies of the general characteristics of the neutralinos in their production and decays [4, 5, 6, 7] as well as in the selectron pair production [8] at e^+e^- and/or e^-e^- linear colliders have been performed.

In the present work, we analyze two–body tree–level decays of neutralinos into a neutralino plus a Z boson or a lightest neutral Higgs boson h in order to probe the Majorana nature of the neutralinos and CP violation in the neutralino system. A comprehensive analysis of the two–body decays of neutralinos as well as charginos was given previously in Ref. [9]. We however note that a rather light Higgs boson mass was assumed and no Z boson polarization was considered in the previous work. One powerful diagnostic tool in the present analysis is Z polarization, which can be reconstructed with great precision through Z–boson leptonic decays, $Z \to l^+l^-$, in particular, with $l = e, \mu$.

It is possible that due to the masses of the relevant particles, no two–body tree–level decays are allowed, in which case the dominant decays would consist of three–body tree–level [10] or two–body one–loop decays [11]. However, a sufficiently heavy neutralino can decay via tree–level two–body channels containing a Z or h with its mass less than 135 GeV in the context of the MSSM [12]. If some sfermions are sufficiently light, two–body tree–level decays of neutralinos into a fermion and a sfermion may be also be important. However, neutralinos heavier than the squarks will be extremely difficult to isolate at hadron colliders, because the squarks and gluinos are strongly produced and they decay subsequently into lighter neutralinos and charginos. On the other hand, at e^+e^- colliders, squarks and sleptons, if they are kinematically accessible, are fairly easy to produce and study directly. With these phenomenological aspects in mind, we assume in the present work that all the sfermions are heavier than (at least) the second lightest neutralino $\tilde{\chi}_2^0$. Then, we investigate

the MSSM parameter space for the two–body tree–level decays of the neutralino $\tilde{\chi}_2^0$ and show how the Majorana nature and CP properties of the neutralinos can be probed through the two–body decays $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 Z$, once such two–body decays are kinematically allowed.

The paper is organized as follows. Section 2 is devoted to a brief description of the mixing for the neutral gauginos and higgsinos in CP–noninvariant theories with non–vanishing phases. In Sec. 3, after explaining the reconstruction of Z–boson polarization through the Z decays into two–lepton pairs, we present the formal description of the (polarized) decay widths of the two–body neutralino decays into a lightest neutralino $\tilde{\chi}_1^0$ plus a Z boson or a lightest Higgs boson h with special emphasis on the polarization of the Z boson. In Sec. 4, we first investigate the region of the MSSM parameter space where the two–body neutralino decays are allowed and discuss the dependence of the branching ratios and decay widths on the relevant SUSY parameters. Then, we give a simple numerical demonstration of how the Majorana nature and CP properties of the neutralinos can be probed through the two–body decays $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 Z$. Finally, we conclude in Sec. 5.

2 Neutralino Mixing

In the MSSM, the mass matrix of the spin-1/2 partners of the neutral gauge bosons, $\tilde{B}$ and $\tilde{W}^3$, and of the neutral Higgs bosons, $\tilde{H}_1^0$ and $\tilde{H}_2^0$, takes the form

$$
\mathcal{M}_N = \begin{pmatrix}
M_1 & 0 & -m_Z c_\beta s_W & m_Z s_\beta s_W \\
0 & M_2 & m_Z c_\beta c_W & -m_Z s_\beta c_W \\
-m_Z c_\beta s_W & m_Z c_\beta c_W & 0 & -\mu \\
m_Z s_\beta s_W & -m_Z s_\beta c_W & -\mu & 0
\end{pmatrix},
\tag{1}
$$

in the $\{\tilde{B}, \tilde{W}^3, \tilde{H}_1^0, \tilde{H}_2^0\}$ basis. Here M_1 and M_2 are the fundamental SUSY breaking U(1) and SU(2) gaugino mass parameters, and μ is the higgsino mass parameter. As a result of electroweak symmetry breaking by the vacuum expectation values of the two neutral Higgs fields v_1 and v_2 ($s_\beta = \sin\beta$, $c_\beta = \cos\beta$ where $\tan\beta = v_2/v_1$), non–diagonal terms proportional to the Z–boson mass m_Z appear and the gauginos and higgsinos mix to form the four neutralino mass eigenstates $\tilde{\chi}_i^0$ ($i = 1$–4), ordered according to increasing mass. In general the mass parameters M_1, M_2 and μ in the neutralino mass matrix (1) can be complex. By re–parameterization of the fields, M_2 can be taken real and positive, while the U(1) mass parameter M_1 is assigned the phase Φ_1 and the higgsino mass parameter μ the

phase Φ_μ. For the sake of our latter discussion, it is worthwhile to note that in the limit of large $\tan\beta$ the gaugino–higgsino mixing becomes almost independent of $\tan\beta$ and the neutralino sector itself becomes independent of the phase Φ_μ in this limit.

The neutralino mass eigenvalues $m_i \equiv m_{\tilde{\chi}_i^0}$ ($i = $ 1-4) can be chosen positive by a suitable definition of the mixing matrix N, rotating the gauge eigenstate basis $\{\tilde{B}, \tilde{W}^3, \tilde{H}_1^0, \tilde{H}_2^0\}$ to the mass eigenstate basis of the Majorana fields: $N^* \mathcal{M}_N N^\dagger = \mathrm{diag}(m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}, m_{\tilde{\chi}_3^0}, m_{\tilde{\chi}_4^0})$. In general the mixing matrix N involves 6 non–trivial angles and 9 non–trivial phases, which can be classified into three Majorana phases and six Dirac phases [4]. The neutralino sector is CP conserving if μ and M_1 are real, which is equivalent to vanishing Dirac phases (mod π) and Majorana phases (mod $\pi/2$). Majorana phases of $\pm\pi/2$ do not signal CP violation but merely indicate different intrinsic CP parities of the neutralino states in CP–invariant theories [7].

3   Two–Body Decays of Neutralinos

Before describing the two–body decays $\tilde{\chi}_i^0 \to \tilde{\chi}_j^0 Z$ in detail, we explain how to reconstruct the Z polarization through the lepton angular distributions of the Z–boson leptonic decays, $Z \to l^- l^+$, particularly with $l = e, \mu$. In the rest frame of the decaying Z boson, which can be reconstructed with great precision by measuring the lepton momenta, the lepton angular distributions are given by

$$\frac{1}{\Gamma[Z \to l^+ l^-]} \frac{d\Gamma[Z(\pm) \to l^+ l^-]}{d\cos\theta_l} = \frac{3}{8} \left[1 + \cos^2\theta_l \pm 2\,\xi_l \cos\theta_l \right] ,$$

$$\frac{1}{\Gamma[Z \to l^+ l^-]} \frac{d\Gamma[Z(0) \to l^+ l^-]}{d\cos\theta_l} = \frac{3}{4}\,\sin^2\theta_l , \tag{2}$$

for the Z–boson helicities, ± 1 and 0, respectively, where $\xi_l = 2 v_l a_l/(v_l^2 + a_l^2) \simeq -0.147$ with $v_l = s_W^2 - 1/4$ and $a_l = 1/4$, and θ_l is the polar angle of the l^- momentum with respect to the Z boson polarization direction. Here, the decay width $\Gamma[Z \to l^+ l^-]$ is the average of three polarized decay widths,

$$\Gamma[Z \to l^+ l^-] = \frac{1}{3} \left\{ \Gamma[Z(+) \to l^+ l^-] + \Gamma[Z(0) \to l^+ l^-] + \Gamma[Z(-) \to l^+ l^-] \right\}. \tag{3}$$

We emphasize that the three polar–angle distributions (2) can be determined without knowing the full kinematics of the decay $\tilde{\chi}_i^0 \to \tilde{\chi}_j^0 Z$. In contrast, the distributions involving the interference of the amplitudes with different Z helicities are always accompanied with

azimuthal angle dependent terms. As the lightest neutralino $\tilde{\chi}_1^0$ assumed to be the lightest SUSY particle (LSP) always escapes detection, the kinematics of the two–body decay $\tilde{\chi}_i^0 \to \tilde{\chi}_j^0 Z$ cannot be fully reconstructed so that the azimuthal–angle dependent distributions are not fully available.

The decay width of the decay $\tilde{\chi}_i^0 \to \tilde{\chi}_j^0 Z$ producing a Z boson with its helicity, ± 1 or 0, reads

$$\Gamma\left[\tilde{\chi}_i^0 \to \tilde{\chi}_j^0 Z(\pm)\right] = \frac{g_Z^2 \lambda_Z^{1/2}}{16\pi m_i^3} (|V|^2 + |A|^2) \left[m_i^2 + m_j^2 - m_Z^2 - 2m_i m_j \, \mathcal{A}_N \pm \frac{\lambda_Z^{1/2}}{2} \mathcal{A}_T \right],$$

$$\Gamma\left[\tilde{\chi}_i^0 \to \tilde{\chi}_j^0 Z(0)\right] = \frac{g_Z^2 \lambda_Z^{1/2}}{16\pi m_i^3} (|V|^2 + |A|^2) \left[\frac{\lambda_Z}{m_Z^2} + m_i^2 + m_j^2 - m_Z^2 - 2\, m_i m_j \, \mathcal{A}_N \right], \quad (4)$$

respectively, where the asymmetries $\mathcal{A}_N$ and $\mathcal{A}_T$ are defined in terms of the vector and axial–vector couplings V and A of the Z boson to the neutralino current as

$$\mathcal{A}_N = \frac{|V|^2 - |A|^2}{|V|^2 + |A|^2}, \qquad \mathcal{A}_T = \frac{2\, \Re e(V A^*)}{|V|^2 + |A|^2}, \quad (5)$$

with $g_Z = g/\cos\theta_W$ and the kinematical factor $\lambda_Z = [(m_i + m_j)^2 - m_Z^2][(m_i - m_j)^2 - m_Z^2]$. Combining the leptonic Z–boson decay distributions (2) with the polarized decay widths (4), we obtain the correlated polar–angle distribution:

$$\begin{aligned}
\frac{d\Gamma_{\text{corr}}}{d\cos\theta_l} = &\frac{3}{8} \mathcal{B}[Z \to ll] \left\{ \left(\Gamma[\tilde{\chi}_i^0 \to \tilde{\chi}_j^0 Z(+)] + \Gamma[\tilde{\chi}_i^0 \to \tilde{\chi}_j^0 Z(-)] \right) (1 + \cos^2\theta_l) \right. \\
&+ 2 \left(\Gamma[\tilde{\chi}_i^0 \to \tilde{\chi}_j^0 Z(+)] - \Gamma[\tilde{\chi}_i^0 \to \tilde{\chi}_j^0 Z(-)] \right) \xi_l \cos\theta_l \\
&\left. + 2\, \Gamma[\tilde{\chi}_i^0 \to \tilde{\chi}_j^0 Z(0)] \sin^2\theta_l \right\}.
\end{aligned} \quad (6)$$

Consequently, each polarized decay width can be extracted from the correlated polar–angle distribution by projecting out the distribution with a proper lepton–polar angle distribution.

The explicit forms of the vector and axial–vector couplings V and A in Eq. (5) are given in terms of the 4×4 neutralino diagonalization matrix N in the MSSM by

$$V = -\frac{i}{2} \Im m \left(N_{j3} N_{i3}^* - N_{j4} N_{i4}^* \right), \qquad A = \frac{1}{2} \Re e \left(N_{j3} N_{i3}^* - N_{j4} N_{i4}^* \right). \quad (7)$$

Note that *the vector coupling V is pure imaginary and the axial–vector coupling A is pure real.* This characteristic property of the Z-$\tilde{\chi}_i^0$-$\tilde{\chi}_j^0$ coupling due to the Majorana nature of neutralinos leads to one important relation between the polarized decay widths with the Z–boson helicities, ± 1:

$$\Gamma[\tilde{\chi}_i^0 \to \tilde{\chi}_j^0 Z(+)] = \Gamma[\tilde{\chi}_i^0 \to \tilde{\chi}_j^0 Z(-)], \quad (8)$$

which is valid even in the CP non–invariant theory. This relation can be checked by measuring the forward–backward polar–angle asymmetry of the correlated polar–angle distribution (6). However, because of the small analyzing power $\xi_l \simeq -0.147$, it will be necessary to have sufficient large number of decay events to measure the asymmetry with good precision. In addition to the relation (8), the relative intrinsic CP parity of two neutralinos in the CP invariant theory can be determined by measuring the ratio of the longitudinal decay width to the transverse decay width, which satisfies

$$\mathcal{R}_{LT} \equiv \frac{2\Gamma[\,\tilde{\chi}_i^0 \to \tilde{\chi}_j^0 Z(0)]}{\Gamma[\,\tilde{\chi}_i^0 \to \tilde{\chi}_j^0 Z(+)] + \Gamma[\,\tilde{\chi}_i^0 \to \tilde{\chi}_j^0 Z(-)]} = \frac{(m_i \mp m_j)^2}{m_Z^2}\,, \tag{9}$$

for the even/odd relative intrinsic CP parity with $V = 0/A = 0$, i.e. $\mathcal{A}_N = \mp 1$, respectively. In the CP non–invariant theory, both the vector and axial couplings are in general non–vanishing, leading to the value of the asymmetry A_N different from ± 1. Therefore, any precise measurements of the asymmetry A_N will provide us with an important probe of CP violation in the neutralino system under the assumption that *the neutralino masses are measured with good precision, independently of the decay modes.*

Next, we give the decay formulas into final states containing a lightest neutral Higgs boson h. The explicit form of the decay width of the decay $\tilde{\chi}_i^0 \to \tilde{\chi}_j^0 h$ is written as

$$\Gamma\left[\tilde{\chi}_i^0 \to \tilde{\chi}_j^0 h\right] = \frac{g^2 \lambda_h^{1/2}}{16\pi m_i^3} \left[|S|^2\left((m_i + m_j)^2 - m_h^2\right) + |P|^2\left((m_i - m_j)^2 - m_h^2\right)\right]\,, \tag{10}$$

with the kinematical factor $\lambda_h = [(m_i + m_j)^2 - m_h^2][(m_i - m_j)^2 - m_h^2]$. The scalar and pseudoscalar couplings, S and P, of the Higgs boson h to the neutralino current are defined in terms of the mixing matrix N as

$$S = \frac{1}{2}\,\Re\mathrm{e}\left[(N_{j2} - t_W N_{j1})(s_\alpha N_{i3} + c_\alpha N_{i4}) + (i \leftrightarrow j)\right]\,,$$
$$P = \frac{i}{2}\,\Im\mathrm{m}\left[(N_{j2} - t_W N_{j1})(s_\alpha N_{i3} + c_\alpha N_{i4}) + (i \leftrightarrow j)\right]\,, \tag{11}$$

where $t_W = \tan\theta_W$, $c_\alpha = \cos\alpha$ and $s_\alpha = \sin\alpha$ for the neutral Higgs mixing angle α. If the charged Higgs boson mass in the MSSM is very large,

$$c_\alpha \to \sin\beta, \qquad s_\alpha \to -\cos\beta\,, \tag{12}$$

This decoupling approximation of the cosine and sine of the mixing angle α is very good if the charged Higgs mass is larger than twice the Z boson mass [13]. For the sake of discussion, we take the decoupling limit in the present work.

4 Numerical Analysis of Two–Body Decays

In some SUSY scenarios, the lightest neutralino $\tilde{\chi}_1^0$ is the LSP and the second lightest neutralino $\tilde{\chi}_2^0$ among the other three neutralino states are expected to be lighter than sfermions and gluino [3]. Then, the two–body decays $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 Z$ or h as well as the two–body decays of the heavier neutralinos $\tilde{\chi}_{3,4}^0$ [9] will constitute the major decay modes of the neutralinos, respectively, once the two–body tree–level decay modes are kinematically allowed. In the following numerical analysis we will ignore all other modes except for the two–body tree–level decays of the neutralinos.

4.1 Branching ratios

For the branching ratio calculations for the two–body decays $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 Z/h$, we assume that all the SUSY parameters are real, M_1 is related to M_2 by the gaugino mass unification condition $|M_1| = (5/3) \tan^2 \theta_W M_2 \approx 0.5\, M_2$ and the Higgs boson mass m_h is 115 GeV. In addition, we assume that the MSSM Higgs system is in the decoupling regime so that the characteristics of the lightest Higgs boson h is similar to the SM Higgs boson to a good approximation [13].

Figure 1 shows the regions of the decays of $\tilde{\chi}_2^0$ on the $\{\mu, M_2\}$ plane. In the region denoted by "three–body decays", no two–body modes are kinematically allowed. In the "Z region" (red–colored), only the two–body decay into a Z is allowed and in the "Z/h region", both the two–body decays $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 Z/h$ are allowed. We divide the "Z/h region" into three parts, according to $\mathcal{B}[Z] \leq 10\%$, $10\% \leq \mathcal{B}[Z] \leq 20\%$ and $\mathcal{B}[Z] \geq 20\%$. (Here, $\mathcal{B}[Z] \equiv \mathrm{Br}[\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 Z]$.) In addition, as a reference, the region excluded by the experimental bound [14] on the lighter chargino mass $m_{\tilde{\chi}_1^\pm} \geq 104$ GeV is displayed by the blue–hatched region.

We first note that, if $M_2 \lesssim 2m_Z$, the mass difference $m_{\tilde{\chi}_2^0} - m_{\tilde{\chi}_1^0}$ is less than m_Z for all μ and the mass difference is very small for $|\mu| \ll M_1, M_2$. So, as clearly shown in Fig. 1 the two–body decay $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 Z$ is allowed only when $2\,m_Z \lesssim M_2 \lesssim 2|\mu|$ under the assumption of the gaugino mass unification condition. In addition, we find from the figure that for the two–body decays the magnitude of μ is required to be larger than about 270 GeV and that, once the two–body Higgs mode $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 h$ is open kinematically, this two–body decay mode dominates in most of the Z/h region. The region where the decay $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 Z$ is appreciable is not symmetric between positive and negative μ in the Z/h region. The branching ratio
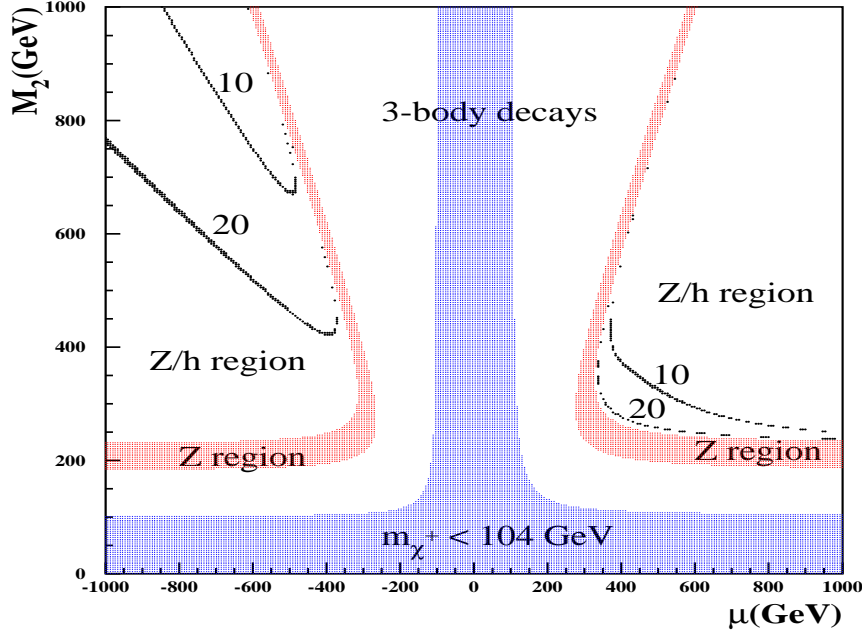
Figure 1: *Three distinct regions of $\tilde{\chi}_2^0$ decay are exhibited as a function of M_2 and μ, assuming that M_2 and μ are real. In the region denoted by "three–body decays", no two–body modes for the neutralino $\tilde{\chi}_2^0$ except for the loop–induced two–body radiative decays are kinematically allowed. In the "Z region" (red–colored), only the two–body decay into a Z is allowed and in the "Z/h region", both the two–body decays are allowed. The "Z/h region" is divided into three parts, according to $\mathcal{B}[Z] \leq 10\%$, $10\% \leq \mathcal{B}[Z] \leq 20\%$ and $\mathcal{B}[Z] \geq 20\%$. For reference, the exclusion region by the experimental bound on the lighter chargino mass bound $m_{\tilde{\chi}_1^\pm} \geq 104$ GeV is displayed by the blue–hatched region. In this numerical illustration, we set $\tan\beta = 10$.*

$\mathcal{B}[Z]$ is significant only in a small area of the positive μ region, but in a large area of the negative μ region.

On the other hand, we find numerically that, for the heavier neutralinos $\tilde{\chi}_{3,4}^0$, the $\{M_2, \mu\}$ region for the two–body decays $\tilde{\chi}_{3,4}^0 \to \tilde{\chi}_1^0 Z/h$ expands drastically. A large region with small $|\mu|$ but large M_2 as well as with small M_2 but large $|\mu|$ also allows for the two–body decays of the heavier neutralinos, $\tilde{\chi}_{3,4}^0 \to \tilde{\chi}_1^0 Z/h$. Only in the wedge–shaped band region of the width of about 100 GeV around the line satisfying the relation $M_2 \approx 2|\mu|$ no two–body decays for the heavier neutralino $\tilde{\chi}_3^0$ are allowed, while the heaviest neutralino can still decay into $\tilde{\chi}_1^0$ and Z/h in the (almost) entire parameter space, possibly except for the region excluded by

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the experimental lighter chargino mass bound.

Consequently, for most of the parameter space of the MSSM the decays of the two heavier neutralinos are dominated by two–body tree–level processes of which the final state consists of a Z boson or h boson together with one of the lighter neutralinos, or a W boson and one of the charginos. Furthermore, the two–body decays of the second lightest neutralino $\tilde{\chi}_2^0$ can be significant in a large region of the parameter space of the MSSM.
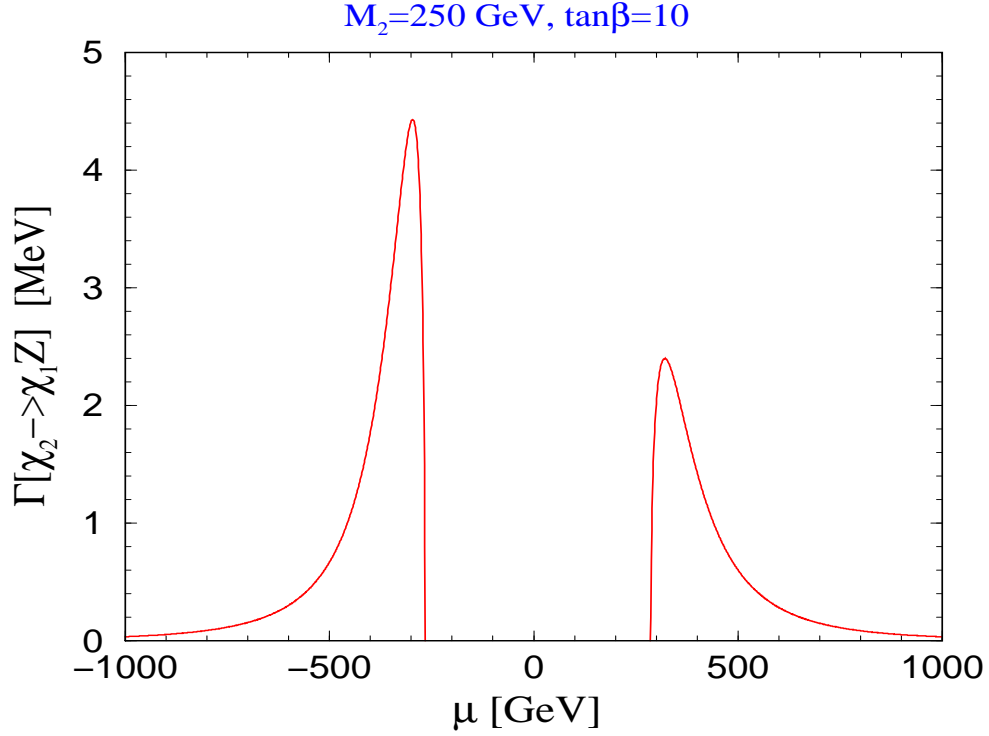

Figure 2: *The dependence of the decay width $\Gamma[\,\tilde{\chi}_2^0 \to \tilde{\chi}_1^0\, Z\,]$ on the higgsino mass parameter μ, assuming that μ is real and $M_1 = (5/3)\, t_W^2\, M_2$. For this numerical illustration, we set $M_2 = 250$ GeV and $\tan\beta = 10$.*

In addition to the branching ratios, it is also crucial to analyze the absolute size of the decay width $\Gamma[\,\tilde{\chi}_2^0 \to \tilde{\chi}_1^0\, Z\,]$. Depending on the values of the relevant couplings, the two–body decay widths could be smaller than the three–body decay widths involving virtual sfermion exchanges, unless the sfermions are too heavy. We exhibit in Fig. 2 the dependence of the decay width $\Gamma[\,\tilde{\chi}_2^0 \to \tilde{\chi}_1^0\, Z\,]$ on the higgsino mass parameter μ, assuming again that μ is real and taking $M_1 = (5/3) \tan^2\theta_W M_2$, $M_2 = 250$ GeV and $\tan\beta = 10$. The decay width

decreases rapidly with increasing $|\mu|$. This is because the couplings of the Z boson to the neutralino current are governed by the higgsino components of the neutralinos (see Eq. (7)) so that the Z-$\tilde{\chi}_i^0$-$\tilde{\chi}_j^0$ couplings are strongly suppressed for large $|\mu|$. Therefore, for large $|\mu|$, some three–body decays could be more dominant than the two–body decays.
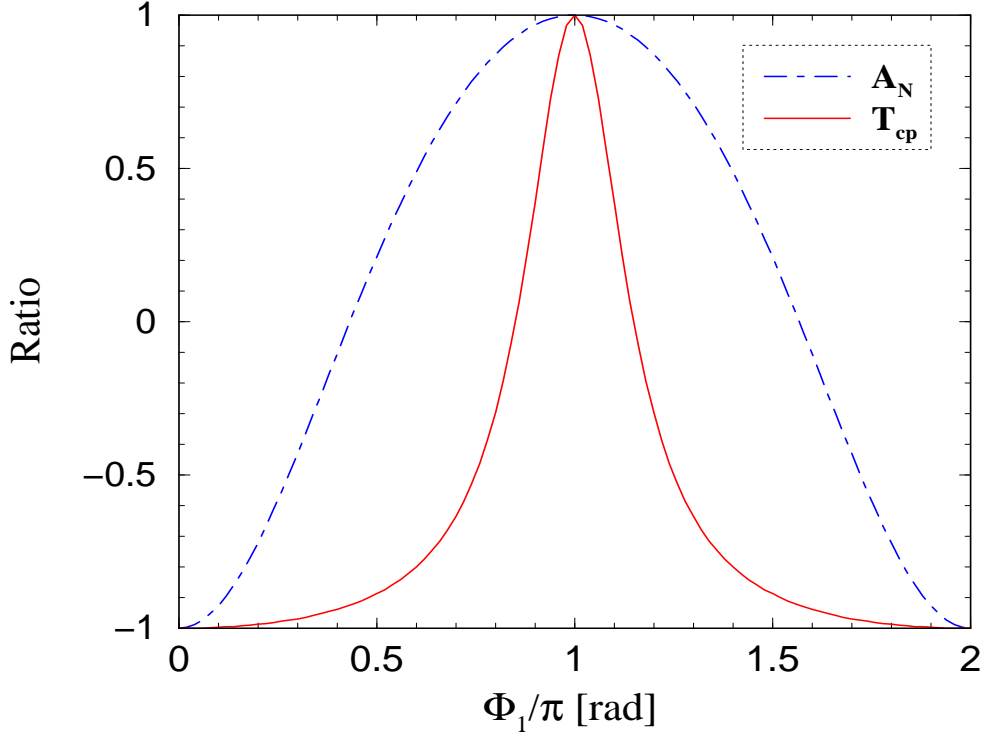

Figure 3: *The dependence of the ratio $\mathcal{T}_{\mathrm{CP}}$ (red solid line) and the asymmetry $\mathcal{A}_N$ (blue dot–dashed line) on the CP phase Φ_1 for the set of real parameters, $\{\tan\beta = 10, M_2 = 250\,\mathrm{GeV}, |\mu| = 500\,\mathrm{GeV}\}$. The phase Φ_μ is set zero in this numerical illustration figure, as the ratio $\mathcal{T}_{\mathrm{CP}}$ and the asymmetry $\mathcal{A}_N$ are found to be insensitive to the phase Φ_μ for the given specific set of real parameters.*

4.2   A probe of CP violation

In the previous subsection, we restrict ourselves to the CP invariant case with real parameters, as the qualitative results obtained from the CP–even quantities are not expected to change so significantly even if the parameters are complex. But, the parameters M_1 and μ are in general complex so that it is important to check whether they indeed have com-

plex phases or not. The existence of the complex phases in the neutralino system, which in general cause CP violation, can be established by the measurements of the ratio

$$\mathcal{T}_{\text{CP}} = \frac{\mathcal{R}_{LT} - (m_i^2 + m_j^2)/m_Z^2}{2\,m_i m_j/m_Z^2}\,, \tag{13}$$

with $\mathcal{R}_{LT}$ defined in Eq. (9) as well as the neutralino masses m_i and m_j. The ratio $\mathcal{T}_{\text{CP}}$ is -1 $(+1)$ in the CP invariant theory for the positive (negative) relative intrinsic CP parity of the neutralinos, $\tilde{\chi}_i^0$ and $\tilde{\chi}_j^0$, taking part in the decay $\tilde{\chi}_i^0 \to \tilde{\chi}_j^0\, Z$, respectively.

To explicitly show the dependence of the ratio $\mathcal{T}_{\text{CP}}$ on the CP phases Φ_1 and Φ_μ, we chose a specific set of real parameters $\{\tan\beta = 10, M_2 = 250\,\text{GeV}, |\mu| = 500\,\text{GeV}\}$ as a simple numerical example with $|M_1| = (5/3)\tan^2\theta_W\, M_2$, while varying the phases Φ_1 and Φ_μ. Numerically, we find that the ratio $\mathcal{T}_{\text{CP}}$ is insensitive to the phase Φ_μ. This is because the phase dependence is always accompanied with $\sin 2\beta = 2\tan\beta/(1 + \tan^2\beta) \approx 0.2$ for $\tan\beta = 10$, which is already small, and the higgsino components of the neutralinos are small for large $|\mu|$. So, we show in Fig. 3 the dependence of the ratio $\mathcal{T}_{\text{CP}}$ (red solid) as well as the asymmetry $\mathcal{A}_N$ (blue dot–dashed) only on the phase Φ_1 for the real parameter set for one fixed value of $\Phi_\mu = 0$. Clearly, in the CP–invariant case with $\Phi_1 = 0, \pi$ or 2π, the absolute magnitude of the ratio $\mathcal{T}_{\text{CP}}$ as well as the asymmetry $\mathcal{A}_N$ is 1, but it is different from 1 in the CP non–invariant case. In the given real parameter set, we find that the ratio $\mathcal{T}_{\text{CP}}$ is quite sensitive to the phase Φ_1 near $\Phi_1 = \pi$, while it is not so sensitive to the phase near $\Phi_1 = 0$ and 2π.

In the present work the analysis for probing the Majorana nature and CP violation in the neutralino system has been carried out at the tree level. However, it will be important to include loop corrections to the two–body tree–level decays because, if they are small, the tree–level CP violation effects might be diluted by loop–induced CP violation effects originating from other sectors of the MSSM.

5 Conclusions

For a large portion of the MSSM parameter space, the decay of the second lightest neutralino $\tilde{\chi}_2^0$ as well as the heavier neutralinos $\tilde{\chi}_{3,4}^0$ could be dominated by two–body processes in which the final state consists of a Z or a lightest Higgs boson h together with a lightest neutralino $\tilde{\chi}_1^0$, assumed to be the lightest supersymmetric particle. The main conclusion

of the present work is that, unless the two–body decay $\tilde{\chi}_i^0 \to \tilde{\chi}_j^0 Z$ is strongly suppressed, the Z polarization, which can be reconstructed through great precision via the leptonic Z–boson decays $Z \to l^+ l^-$, provides us with a powerful probe of the Majorana nature of the neutralinos and CP violation in the neutralino system.

Acknowledgments

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